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CALCULATING THE SPEED WITH WHICH WATER-PERMEABLE GROUND IS FROZ--ETC(U)
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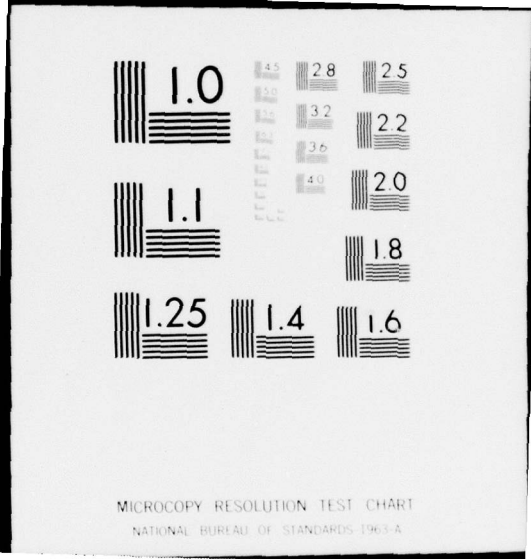
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CALCULATING THE SPEED WITH WHICH
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BY A ROW OF COLUMNS BEFORE
FROZEN GROUND CYLINDERS JOIN

A.I. Pekhovich



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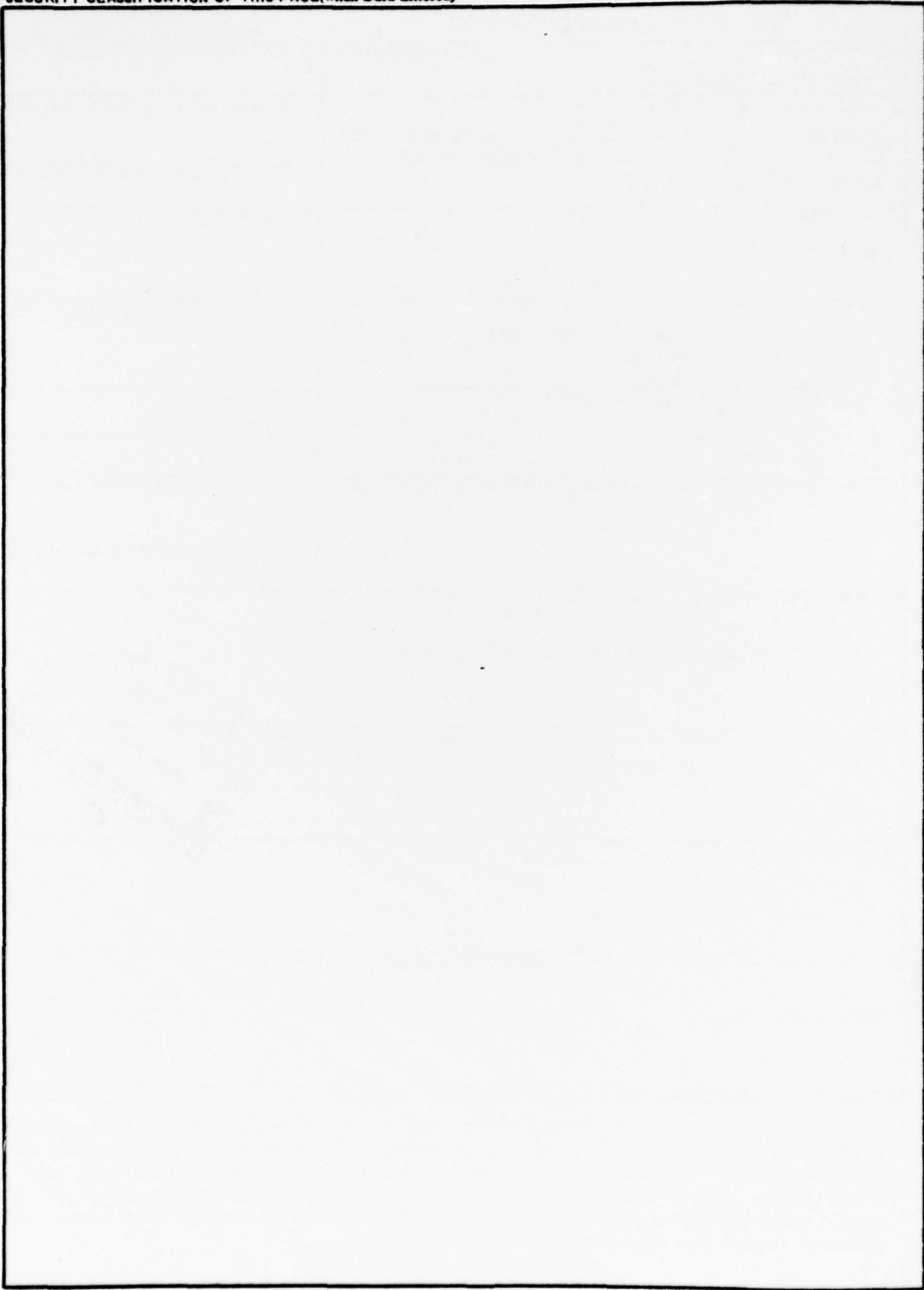
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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER Draft Translation 590	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) CALCULATING THE SPEED WITH WHICH WATER-PERMEABLE GROUND IS FROZEN BY A ROW OF COLUMNS BEFORE FROZEN GROUND CYLINDERS JOIN		5. TYPE OF REPORT & PERIOD COVERED
7. AUTHOR(s) A.I. Pekhovich		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS U.S. Army Cold Regions Research and Engineering Laboratory Hanover, New Hampshire		8. CONTRACT OR GRANT NUMBER(s)
11. CONTROLLING OFFICE NAME AND ADDRESS		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE February 1977
		13. NUMBER OF PAGES 17
		15. SECURITY CLASS. (of this report)
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) FROZEN GROUND		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) In 1954 Volume 51 of the "Notes of the All-Union Scientific Research Institute of Hydraulic Engineering" contained an article which examined the method for calculating the freezing rate of water-permeable ground by a row of columns after frozen ground cylinders had joined. This article proposes a method for calculating the rate at which water-permeable ground is frozen by a series of columns before the frozen ground cylinders join.		

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DRAFT TRANSLATION 590

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ENGLISH TITLE: CALCULATING THE SPEED WITH WHICH WATER-PERMEABLE GROUND IS FROZEN BY A ROW OF COLUMNS BEFORE FROZEN GROUND CYLINDERS JOIN

FOREIGN TITLE: (RASCHET SKOROSTI ZAMORAZHIVANIYA FIL'TRUYUSHCHEGO GRUNTA RYADOM KOLONOK DO SMYKANIYA LEDOGRUNTOVYKH TSILINDROV)

9
AUTHOR: A. I. / Pekhovich

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SOURCE: Trans. of Izvestiya Vsesoyuznogo Nauchno-Issledovatel'skogo Instituta Gidrotekhniki imeni B. Ye. Vedeneyeva Tom 58, 1958, p.187-200. (USSR) V58 p187-200 198.

Translated by Office of the Assistant Chief of Staff for Intelligence for U.S. Army Cold Regions Research and Engineering Laboratory, 1977, 17p.

14
12/24p.
CRREL-TL-594

11 Feb 77
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In 1954 Volume 51 of the "Notes of the All-Union Scientific Research Institute of Hydraulic Engineering" contained an article which examined the method for calculating the freezing rate of water-permeable ground by a row of columns after frozen ground cylinders had joined. This article proposes a method for calculating the rate at which water-permeable ground is frozen by a series of columns before the frozen ground cylinders join.

1. Statement of the Problem

The problem under examination can be formulated as follows.

Let there be given a cofferdam of length L and thickness H (Figure 1). The cofferdam is built on a base which is permeable to water and heat. Water filters through the cofferdam under the action of a constant difference in head between the lateral boundaries of the cofferdam. Water does not filter around the cofferdam. The freezing columns with radius r_0 , height h and total number L are arranged on the cofferdam on a plane which is parallel to the lateral (head) surfaces at uniform distances S from each other. Before the columns are implanted in the ground, the filtration flow under natural (normal) conditions has speed v_n and temperature T . The ground freezes at temperature t_0 . The thermophysical parameters of the frozen and unfrozen ground are known. It is necessary to find a dependency which determines the ground freezing rate before the cylinders join.

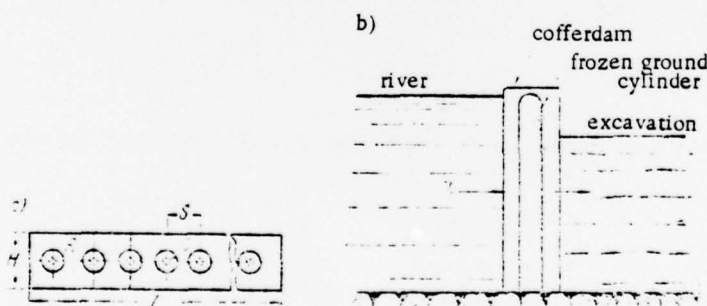


Figure 1. a, Plan of Cofferdam; b, Transverse Cross-Section of Cofferdam.

2. Heat Balance Equation

The heat balance equation of the frozen ground looks as follows:

$$Q_f = Q_c - Q_{fi} - Q_g, \quad (1)$$

where Q_f is the amount of heat evolved per unit of time when water freezes on the surface of the frozen ground cylinders;

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Q_c is the amount of heat which is given off per unit of time by the freezing columns (the thermal absorption by the freezing columns);

Q_{fi} is the amount of heat which the filtration flow gives off per unit of time to the frozen ground cylinders;

Q_g is the amount of heat which the frozen ground loses per unit of time (cooling of the frozen ground cylinders).

The thermal flow Q_g , as is demonstrated in examining the freezing of water-permeable ground by a single column [1] can be taken into account by increasing the calculated value of the latent ice formation heat by the value

$$\tau_g = \frac{c_{f1} Q_c}{4\pi\lambda_1 h} \quad (2)$$

where λ_1 is the thermal conductivity factor of the frozen ground;

c_{f1} is the thermal capacity of the frozen ground;

γ_1 is the specific weight of the frozen ground;

h is the height of the frozen ground cylinder.

Therefore thermal balance equation (1) can be represented in the following simpler form:

$$Q_f = Q_c - Q_{fi} \quad (3)$$

The thermal flux from the latent ice formation heat is equal to:

$$Q_f = 2\pi R \sigma \tau \frac{dR}{dt} \quad (4)$$

Here R is the radius of the frozen ground cylinder;

σ is the latent ice formation heat of a unit of ground volume;

τ is time.

Allowing for the thermal flux Q_g , instead of (4) it is necessary to write

$$Q_f = 2\pi R \sigma \tau \frac{dR}{dt} \quad (5)$$

where

$$\tau = \tau_g + \tau_f \quad (6)$$

The absorption of heat by the freezing columns changes as the ground freezes. The law governing the change in heat absorption is determined by the characteristic of the refrigeration device and by the thermal resistance of the frozen ground.

The means for determining this dependency are analyzed in the technical literature [2]: here we will write this dependency with regard to the frozen ground cylinders in the following general form:

$$Q_c = f(R).$$

This expression may be regarded as a known dependency; in doing this we keep in mind the fact that by properly selecting the refrigeration devices this dependency can be given any desired form.

If the freezing is conducted at a constant brine temperature θ , then

$$Q_c = \frac{2\pi h \lambda_1 (t_0 - \theta)}{\ln \frac{R}{r_0}}. \quad (7)$$

It remains for us to determine the value of the thermal influx from the filtration flow Q_{fi} . The determination of this component of heat balance equation (3) is the basic difficulty in solving the stated problem.

3. Calculating the Thermal Influx from the Unfrozen Water-Permeable Ground to the Frozen Ground Cylinders

We will determine the value of the thermal flux from the unfrozen ground to the frozen ground cylinders on the basis of the following equation:

$$Q_{fi} = c_w \gamma_w S h I v_0 (T - t_{out}),$$

where c_w is the thermal capacity of the water;

γ_w is the specific weight of the water;

I is the number of columns;

v_0 is the speed of the filtration flow in the cofferdam in the area where the frozen ground cylinders do not cover the entire cross-section;

S is the distance between the axes of the freezing columns;

T is the temperature of the filtration flow at the approach to the frozen ground cylinders (equal to the temperature of the filtration flow under natural conditions);

t_{out} is the average temperature of the filtration flow after passing through the frozen ground screen.

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In order to make subsequent analysis convenient, we will introduce two parameters:

$$\nu = \frac{v_0}{v_b}, \quad (8)$$

where v_b is the speed of the filtration flow before freezing begins and

$$\Delta \bar{t} = \frac{T - t_{out}}{T - t_0}; \quad (9)$$

where as before t_0 is the freezing temperature of the ground.

Then the equation for the thermal flux Q_{fi} acquires the following form:

$$Q_{fi} = N \Delta \bar{t}, \quad (10)$$

where

$$N = \frac{c \gamma S h l v_b}{w w} (T - t_0). \quad (11)$$

In order to use equation (10) it is necessary to know the values of ν and $\Delta \bar{t}$.

In the problem which we are examining (a row of freezing columns, without filtration around the cofferdam) as the frozen ground cylinders increase in size, the rate of flow through the cofferdam decreases, and therefore the speed of the filtration flow v_0 and the value of ν decrease. This problem was studied by G. S. Shadrin who used the graphic method of constructing motion grids [3]. An analytic examination of this problem has led us to the following relationship [4] which determines the desired parameter:

$$\nu = \frac{v_0}{v_b} = \frac{z}{z - (\Phi_i - z)}, \quad (12)$$

where

$$z = \frac{2R}{S}; \quad (13)$$

$$z = \frac{H}{S}; \quad (14)$$

$$\Phi_i = \frac{\arcsin z + \frac{\pi}{2}}{1 - z^2} - \frac{\pi}{2}. \quad (15)$$

A graph of the dependency $\nu = f(\varepsilon, \chi)$ is presented in Figure 2.

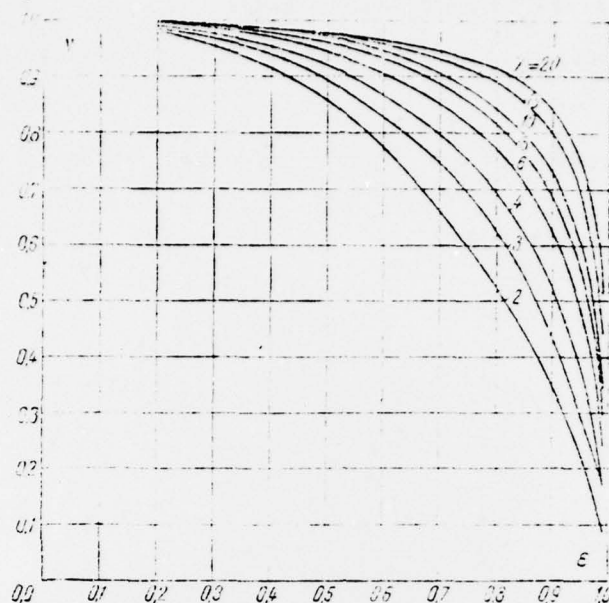


Figure 2. Graph for Determining the Relative Change in Filtration Flow Through the Cofferdam When the Frozen Ground Cylinders Increase in Size.

Let us now determine $\Delta \bar{t}$. For this purpose we will use the solution given by heat transmission theory for the problem of wall cooling [5]. The problem is formulated as follows.

An infinite, homogeneous wall of thickness $2X$ at the initial time ($\tau = 0$) has the same temperature T at all points, and then ($\tau > 0$) both sides of its surface are exposed to a temperature of t_0 . The temperature conductivity factor of the wall a is known.

The solution to this problem has the following form:

$$\Delta \bar{t} = f(\text{Fo}), \quad (16)$$

where

$$\text{Fo} = \frac{a\tau}{X^2}. \quad (17)$$

A graph of dependency (16) is presented in Figure 3, the lower left quadrant (see glued-in piece at the end of the book).

We will show how to use this solution in our case.

We will write the differential equation of heat conductivity in water-permeable ground (the arrangement of the coordinate axes is given in Figure 4):

$$c_w w \left(v_x \frac{\partial t}{\partial x} + v_y \frac{\partial t}{\partial y} \right) = \rho_w \left(\frac{\partial t}{\partial \tau} + \frac{\partial t}{\partial y^2} \right). \quad (18)$$

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here v_y is the projection of the filtration flow's velocity vector onto axis Y,

v_x is the projection of the filtration flow's velocity vector onto axis X.

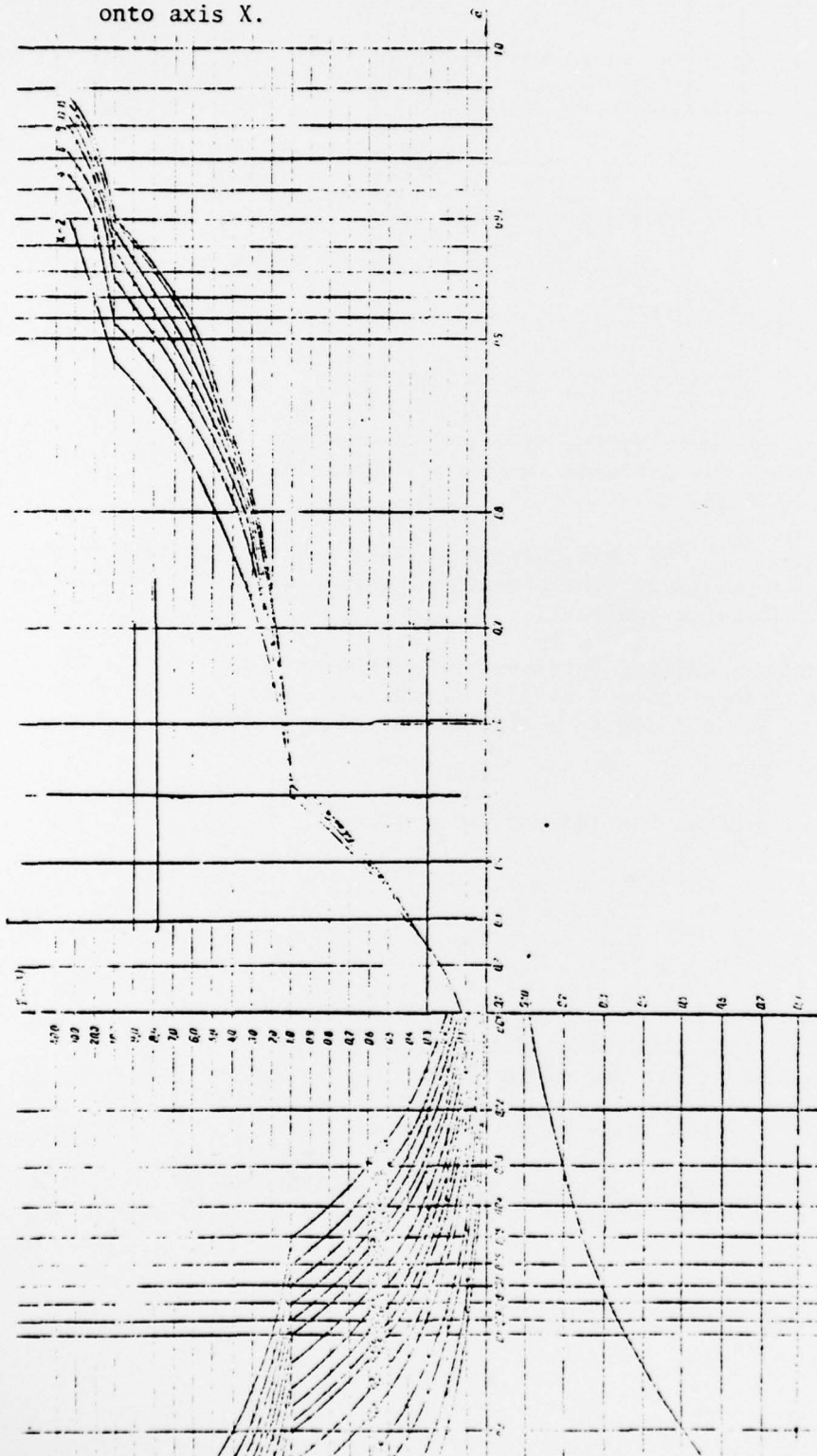


Figure 3.

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Ignoring the value of the thermal flux which arises due to physical heat conductivity and is directed along axis Y and the component of induced convection which is directed along X, we will convert equation (18) into the following form:

$$\frac{\partial t}{\partial y} = \frac{a}{v_x} \frac{\partial^2 t}{\partial x^2}, \quad (19)$$

where

$$a = \frac{\lambda}{\rho W} \quad (20)$$

λ_2 is the thermal conductivity factor of unfrozen ground.

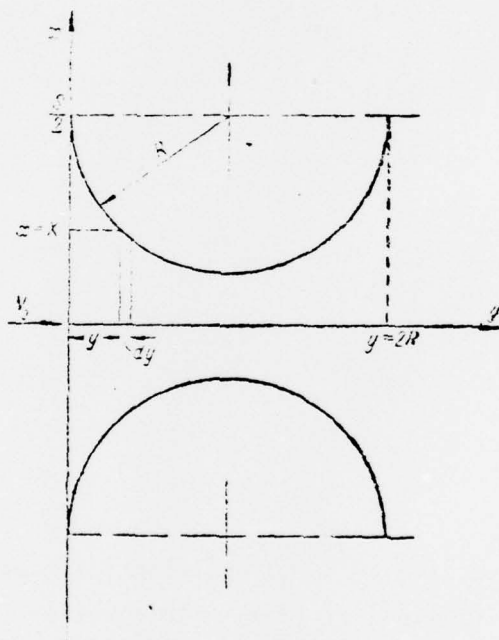


Figure 4.

Equation (19) can be regarded as a one-dimensional thermal conductivity equation in which the time variable is replaced by the y coordinate variable.

Thus, the task of cooling a filtration flow which occurs in a gap $2X$ thick amounts to the task of cooling a wall, the solution to which is known (equation (16)). However, in the case which is of interest to us, cooling a filtration flow when it passes through a frozen ground stream, we are dealing with a gap of non-constant thickness.

We will show that the above-indicated solution can also be applied to our problem.

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As is known, the temperature distribution in an infinite planar-parallel wall is determined by integrating the differential equation

$$\frac{\partial t}{\partial \tau} = a \frac{\partial^2 t}{\partial X^2} \quad (21)$$

If we place the origin of the coordinate axis in the middle of the wall ($x = 0$) and designate the wall thickness as $2X$, then under boundary conditions

$$t(X, 0) = T, \quad (22)$$

$$t(-X, \tau) = t(X, \tau) = t_m; \quad (23)$$

the solution to equation (21), as is well known, will be as follows [5]:

$$t(\tau, \tau) = t_m + (T - t_m) \sum_{n=1}^{\infty} A_n \cos(\mu_n \tau) \exp(-\mu_n^2 Fo) \quad (24)$$

where

$$\mu_n = \frac{\lambda_n}{X}; \quad (25)$$

$$Fo = \frac{a\tau}{X^2}; \quad (26)$$

$$\mu_n = (2n-1) \frac{\pi}{2}; \quad (27)$$

$$A_n = \frac{4}{\pi(2n-1)} \exp(-\mu_n^2). \quad (28)$$

For any moment in time ($\tau = \tau_0$) equation (24) can be regarded as an initial condition (instead of (22)) with equation (21) integrated. This initial condition is characterized completely by the value of criterion Fo ; in this case, from the viewpoint of the subsequent process it is completely irrelevant what the conditions are under which the process of heat transmission has occurred during the preceding period. The course of the wall's temperature, as before, will be described by equation (24), and the speed of increase of Fo will depend on the wall thickness at the given time. We will consider that the change in wall thickness and, consequently, the change in the area of the filtration flow's cross-section per se does not cause any temperature changes at compatible points, i.e., we will assume that when the stream of the filtration flow is instantaneously compressed or expanded, dependency $t = f(\gamma_1)$ will remain unchanged.

In this case

$$\frac{\partial Fo}{\partial X} = 0. \quad (29)$$

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And, consequently,

$$dFo = \frac{\partial Fo}{\partial \tau} d\tau. \quad (30)$$

Therefore the value of criterion Fo which has to be substituted into the solution of (24) is determined by the integral:

$$Fo = a \int_0^{\tau} \frac{d\tau}{X^2}. \quad (31)$$

In order to obtain this integral, we will express $d\tau$ through X , and in doing so we note that

$$d\tau = \frac{dy}{v_x}. \quad (32)$$

From Figure 4 it follows

$$y = R - \sqrt{R^2 - \left(\frac{S}{2} - X\right)^2}.$$

by differentiation, we find:

$$dy = \frac{X - \frac{S}{2}}{\sqrt{R^2 - \left(\frac{S}{2} - X\right)^2}} dX. \quad (33)$$

Keeping in mind the fact that the average velocity of the filtration flow is inversely proportional to the area of the cross-section, we can write:

$$v_x = v \frac{S}{2X}. \quad (34)$$

By substituting (32) into (31) and taking (33) and (34) into account, we find:

$$Fo = \frac{2a}{vS} \int_0^{\tau} \frac{X - \frac{S}{2}}{X \sqrt{R^2 - \left(\frac{S}{2} - X\right)^2}} dX.$$

By integrating within the limits of $X = \frac{S}{2}$ to $X = \frac{S}{2} - R$, doubling the result (since the limits cover only half of the length of the slit) and by substituting expression (15), we find the value of the Fourier criterion after the filtration flow passes through the frozen ground screen:

$$Fo = \frac{2a}{vS} \Phi_1.$$

and, keeping (12) in mind, we finally obtain:

$$Fo = K^2 \psi(z, \varepsilon), \quad (35)$$

where

$$\psi(z, \varepsilon) = \frac{z + \psi_1 - \varepsilon}{z} \psi_1; \quad (36)$$

$$K = \frac{4a}{c_b S}; \quad (37)$$

ψ_1 -- see equation (15).

The results of calculating the Fourier criterion in terms of dependency (35) are also presented in Figure 3. By knowing χ , ε and K , this dependency makes it possible to find $\Delta \bar{t}$.

Thus, by using the graph in Figure 2 it is possible to determine the values of parameter ν , and from the nomogram in Figure 3 it is possible to find the values of parameter $\Delta \bar{t}$. This is sufficient to calculate the magnitude of the heat influx from the unfrozen water-permeable ground according to formula (10). Below is given a corresponding sample calculation for the purpose of illustration.

4. Sample Calculation of Heat Influx From Unfrozen Water-Permeable Ground

We will determine the thermal influx to one of the frozen ground cylinders which is in a row with others.

The following are given: $\lambda_2 = 1 \frac{\text{kcal}}{\text{m} \cdot \text{hr} \cdot \text{deg}}$;

$c_w = 1 \frac{\text{kcal}}{\text{m} \cdot \text{deg}}$;

$\gamma_w = 1000 \frac{\text{kg}}{\text{m}^3}$;

$\nu_b = 0.04 \text{ m/hr}$; $T = 7^\circ\text{C}$; $t_0 = 0^\circ\text{C}$; $S = 1 \text{ m}$; $H = 3 \text{ m}$; $I = 1$; $h = 1 \text{ m}$.

From equations (20), (14), (37), and (11), we find:

$$a = \frac{\lambda_2}{c_w \gamma_w} = \frac{1}{1 \cdot 1000} = 0.001 \text{ m}^2/\text{hr};$$

$$\chi = \frac{H}{S} = \frac{3}{1} = 3;$$

$$K = \frac{4a}{c_b S} = \frac{4 \cdot 0.001}{0.04 \cdot 1} = 0.1;$$

$$N = c_w S h \nu_b (T - t_0) = 1 \cdot 1000 \cdot 1 \cdot 1 \cdot 0.04 \cdot 7 = 280 \text{ kcal/hr.}$$

According to computation equation (10):

$$Q_{fi} = N \Delta T = 250 \Delta T$$

The rest of the computations are summarized in the table. For the sake of comparison the last graph in the table shows the results of calculating the heat influx to a single frozen ground cylinder, with these computations carried out according to formula of B. V. Proskuryakov [6]:

$$Q_{fi} = 8 \sqrt{\frac{\lambda_0 G_w W_0 S}{2\pi}} k (T - t_0) \sqrt{\epsilon}. \quad (38)$$

TABLE 1.

	Found from Figure 2	ΔT Found from Figure 3	Q_{fi} , kcal/hr	
			Row of columns from formula (10)	Single column according to formula (38)
0.1	0.09	0.15	35.0	44.7
0.2	0.09	0.17	47.1	59.0
0.3	0.072	0.225	61.5	77.5
0.4	0.04	0.28	73.5	89.7
0.5	0.82	0.35	87.2	109.1
0.6	0.825	0.42	97.2	118.0
0.7	0.74	0.55	116.1	123.0
0.8	0.63	0.715	129.0	124.0
0.9	0.457	0.9	113.5	127.5
0.95	0.33	0.95	84.1	127.5
0.98	0.29	1.0	59.0	129.5

Commas indicate decimal points.

From Table 1 it is evident that the magnitude of the heat influx from the unfrozen ground to a cylinder which is in a row with another reaches its peak value when $\epsilon \approx 0.8$, and then rapidly decreases. A comparison with the data for a single cylinder shows that in the cited case conditions for ground freezing by a row of columns are more favorable. The corresponding calculations, however, show convincingly that under these circumstances, for instance at higher filtration flow speeds or when the cofferdam is quite thick, cases are possible where the thermal influx Q_{fi} to the frozen ground cylinders in a row with other cylinders is greater than the flow to a single cylinder.

5. Solution to the Heat Balance Equation

We will substitute into heat balance equation (3) the expressions of its components (5) and (10), and by taking (13) into account we find:

$$\frac{\pi}{2} S k H^2 \frac{dz}{dz} = Q_0 - N \Delta T.$$

The solution to the posed problem, determining the ground freezing time before the frozen ground cylinders join, is found by integrating this latter equation within the limits from $\epsilon = 2r_0/S$ to $\epsilon = 1$ and from $\tau = 0$, to $\tau = \tau_1$:

$$\tau_1 = \frac{\pi}{2} \frac{S \sqrt{hT}}{Q_c - N \sqrt{hT}} \quad (39)$$

The sub-integral function is complex in form, and therefore the value of the integral must be calculated by the graphic or tabular method, and then determining the freezing time according to formula (39) presents no problem.

6. Sample of Calculating Freezing Rate

We will determine the period of ground freezing which occurs before the frozen ground cylinder join under the following initial conditions:

length of cofferdam.....	$L = 100 \text{ m}$
thickness of cofferdam.....	$H = 6 \text{ m}$
height of column.....	$h = 8 \text{ m}$
distance between column axes	$S = 1 \text{ m}$
radius of freezing columns.....	$r_0 = 0.05 \text{ m}$
speed of filtration flow.....	$v_b = 0.08 \text{ m/hr}$
temperature of filtration flow	$T = 6^\circ\text{C}$
latent ice formation heat per unit of ground volume.....	$\sigma = 24,000 \text{ kcal/m}^3$
thermal capacity of water	$c_w = 1 \text{ kcal/kg/hr}$
specific weight of water.....	$\gamma_w = 1000 \text{ kg/m}^3$
thermal conductivity factor of unfrozen ground.....	$\lambda_2 = 1 \text{ kcal/m}\cdot\text{hr}\cdot\text{deg}$
thermal conductivity factor of frozen ground.....	$\lambda_1 = 2 \text{ kcal/m}\cdot\text{hr}\cdot\text{deg}$
thermal capacity of frozen ground.....	$c_1 = 0.34 \text{ kcal/kg}^\circ\text{C}$
specific weight of frozen ground.....	$\gamma_1 = 1600 \text{ kg/m}^3$

We find

$$a = \frac{\lambda_2}{c_w \gamma_w} = \frac{1}{1 \cdot 1000} = 0.001 \text{ m}^2 \text{ hr};$$

$$K = \frac{4a}{v_b S} = \frac{4 \cdot 0.001}{0.08 \cdot 1} = 0.05;$$

$$z = \frac{H}{S} = \frac{6}{1} = 6;$$

$$I = \frac{L}{S} = \frac{100}{1} = 100;$$

$$N = c_w \gamma_w S h v_b (T - t_0) = 1 \cdot 1000 \cdot 1 \cdot 8 \cdot 100 \cdot 0.08 \cdot 6 = 384000 \text{ kcal/hr.}$$

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The refrigeration device ensures the constant brine temperature $\theta = -20^{\circ}\text{C}$; according to formula (7) and taking into account the fact that $R = \frac{2\varepsilon}{S}$, we will determine how much heat the freezing columns absorb:

$$Q_c = \frac{2\pi h l \lambda_1 (t_0 - t_1)}{\ln \frac{S_2}{2r_0}} = \frac{2\pi \cdot 8 \cdot 100 \cdot 2 \cdot 20}{\ln \left(\frac{12}{2 \cdot 0,05} \right)} = \frac{201 \cdot 10^3}{\ln(102)}.$$

The calculated value of the latent ice formation heat is determined from equations (2) and (6):

$$\tau = \tau_0 + \frac{c_{fi} Q_c}{4\pi \lambda_1 l h} = 24000 + \frac{0,34 \cdot 1600}{4\pi \cdot 2 \cdot 100 \cdot 8} Q_c = 24000 + 27 \cdot 10^{-3} Q_c.$$

The rest of the calculations are summarized in Table 2, and in this process, in accordance with equation (39) we utilize the following expression:

$$\Delta \tau = \frac{\pi}{2} S^2 h l \tau' = \frac{\pi \Delta z}{Q_c - N \Delta \tau} = \frac{\pi}{2} l^2 \cdot 8 \cdot 100 \tau' = \frac{\pi \Delta z}{Q_c - N \Delta \tau} = 1260 \tau' = \frac{\pi \Delta z}{Q_c - N \Delta \tau}.$$

TABLE 2.

ε	θ	Q_c , kcal hr	Found from Figure 2	τ Found from Figure 3	$Q_{fi} - N \Delta \tau$ kcal hr	Q_c fi kcal hr	$\frac{Q_c}{Q_{fi} - N \Delta \tau}$ hr kcal	$\Delta \tau$ kcal m ³	According to formula (39), hrs	hrs
0.2	0.1	290 · 10 ³	0.59	0.115	43700	245300	0.81 · 10 ⁻³	31830	3.24	3.24
0.3	0.1	183 · 10 ³	0.59	0.150	60800	122200	2.45 · 10 ⁻³	28241	8.91	12.15
0.4	0.1	145 · 10 ³	0.97	0.195	73900	72000	5.55 · 10 ⁻³	27929	10.44	31.59
0.5	0.1	125 · 10 ³	0.94	0.240	89500	38500	13 · 10 ⁻³	27379	41.8	76.39
0.6	0.1	112 · 10 ³	0.91	0.290	101300	10700	56 · 10 ⁻³	27000	190.5	266.89
0.62	0.02	110 · 10 ³	0.50	0.295	101900	8100	15.3 · 10 ⁻³	26970	52.0	318.89
0.65	0.03	107 · 10 ³	0.58	0.330	111500	0	-	26900	-	-

Commas indicate decimal points.

Thus, under the given conditions joining will not occur; the frozen ground cylinders will cease to grow as early as $\varepsilon \cong 0.64$. In the given case it is advisable to shift to freezing at a lower brine temperature. Below Table 3 shows a calculation of freezing time which demonstrates that if we assume that the brine temperature is $\theta = -40^{\circ}\text{C}$, then joining will occur, for instance, on the 16th day.

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TABLE 3.

		Q_c kcal hr	Found from Figure 2	$\bar{\sigma}$ Found from Figure 3	$Q = N \cdot \bar{\sigma}$ kcal hr	$Q - Q_c$ kcal hr	$\frac{2r_c}{Q - Q_c}$ hr kcal	$\bar{\sigma}$ kcal m ³	According to formula (39), hrs	hrs
0.2	0.1	580 · 10 ⁻³	0.99	0.115	43700	536300	0.37 · 10 ⁻⁷	39700	1.85	1.85
0.3	0.1	366 · 10 ⁻³	0.99	0.160	60500	305200	0.98 · 10 ⁻⁷	23900	4.20	6.05
0.4	0.1	290 · 10 ⁻³	0.97	0.195	73000	217000	1.84 · 10 ⁻⁷	31850	7.36	13.41
0.5	0.1	250 · 10 ⁻³	0.94	0.240	86500	163500	3.05 · 10 ⁻⁷	30750	11.8	25.21
0.6	0.1	224 · 10 ⁻³	0.91	0.290	101300	122700	4.8 · 10 ⁻⁷	30050	18.12	43.33
0.7	0.1	206 · 10 ⁻³	0.85	0.355	116000	90000	7.78 · 10 ⁻⁷	29550	28.95	72.28
0.8	0.1	194 · 10 ⁻³	0.77	0.450	136000	58000	13.8 · 10 ⁻⁷	29230	50.64	112.92
0.9	0.1	184 · 10 ⁻³	0.62	0.690	164000	20000	45 · 10 ⁻⁷	28970	164.0	286.92
0.95	0.05	178 · 10 ⁻³	0.47	0.890	160600	17400	23.8 · 10 ⁻⁷	28800	86.2	373.22
0.98	0.03	176 · 10 ⁻³	0.33	0.995	126000	50000	5.88 · 10 ⁻⁷	28750	21.35	194.57

Commas indicate decimal points.

7. Determining the Joining Conditions

The condition for the cylinders to join into a single frozen ground mass consists of the fact that the heat absorbed by the columns Q_c at all values of ϵ must exceed the thermal influx from the filtration flow, i.e., the following condition must be fulfilled:

$$Q_c > Q_{fi} \quad (40)$$

when

$$\frac{2r_c}{S} \leq 1:$$

Instead of ensuring that condition (40) is fulfilled from the beginning of freezing until the frozen ground cylinders join, as a rule, it is sufficient to ensure that this equality is observed at frozen ground cylinder dimensions which correspond to the maximum Q_{fi} value. Figure 5 shows the graph which makes it possible to approximately determine the value of parameter ϵ at which Q_{fi} reaches its peak. This graph was plotted as follows.

By utilizing Figures 2 and 3, dependencies such as $\Delta \bar{t} \cdot v = f(\epsilon)$ were found for various values $K = \text{const}$ and $\chi = \text{const}$, and each of these curves was used to determine ϵ values which correspond to a maximum value of $\Delta \bar{t} \cdot v$, and consequently to the maximum Q_{fi} value (at given values K and χ). These additional calculations and structures are not given here, but only their results are shown in Figure 5.

We will cite an example of ensuring that the solidification conditions are fulfilled.

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Assume that it is necessary to determine the distance between the freezing columns at which joining of the frozen ground cylinders will be ensured, if it is known that:

length of cofferdam..... $L = 50$ m
 thickness of cofferdam..... $H = 8$ m
 height of freezing columns..... $h = 10$ m
 radius of freezing columns..... $r_0 = 0.05$ m
 speed of filtration flow..... $v_b = 0.05$ m/hr
 temperature of filtration flow..... $T = 5^\circ\text{C}$
 thermal capacity of water..... $c_w = 1$ kcal/kg·deg
 specific weight of water..... $\gamma_w = 1$ kg/m³
 thermal conductivity factor of
 frozen ground..... $\lambda_1 = 2$ kcal/m·hr·deg
 thermal conductivity factor of
 unfrozen ground..... $\lambda_2 = 1$ kcal/m·hr·deg
 brine temperature in columns..... $\theta = -22^\circ\text{C}$
 freezing temperature of ground..... $t_0 = 0^\circ\text{C}$.

According to (11) and bearing in mind the fact that $I = L/S$, we can write

$$N = c_w \gamma_w h L v_b (T - t_0) = 1 \cdot 1000 \cdot 10 \cdot 50 \cdot 0.05 \cdot 5 = 125000 \text{ kcal/hr.}$$

If we assume that $S = 1.5$ m, then according to expression (37) and (14) [with allowance for (20)]:

$$K = \frac{4 \cdot \lambda_2}{c_w \gamma_w h S} = \frac{4 \cdot 1.0}{1 \cdot 1000 \cdot 0.05 \cdot 1.5} = 0.053;$$

$$z = \frac{H}{S} = \frac{8}{1.5} = 5.3.$$

From Figure 5 we determine that the maximum heat influx of the filtration flow occurs when $\varepsilon \approx 0.94$.

From Figures 2 and 3 we find that when $\varepsilon \approx 0.94$:

$v = 0.48$ and $\Delta \bar{t} = 0.86$, and consequently according to (10):

$$Q_{fi} = N \cdot \Delta \bar{t} = 125000 \cdot 0.48 \cdot 0.86 = 51600 \text{ kcal/hr.}$$

On the other hand, the heat absorbed by the columns is equal to:

$$Q_c = \frac{2\pi \lambda_1 (t_0 - \theta)}{\ln \frac{r_0}{2r_c}} = \frac{2\pi \cdot 2 \cdot 10 \cdot 50 \cdot 22}{1.3 \ln \frac{0.94 \cdot 1.5}{2 \cdot 0.05}} = 34850 \text{ kcal/hr.}$$

Thus, it has been found that when $S = 1.5 \text{ m}$ $Q_{fi} > Q_c$ and consequently joining does not occur; therefore it is necessary to set the columns closer together. If we assume that $S = 0.8 \text{ m}$ and repeat the course of the above-cited calculation, we find that the position $\varepsilon = 0.92$ will be most difficult and in this case: $Q_{fi} = 75,000 \frac{\text{kcal}}{\text{hr}}$ and $Q_c = 86,500 \frac{\text{kcal}}{\text{hr}}$. Therefore this version is technically feasible since condition (40) is maintained.

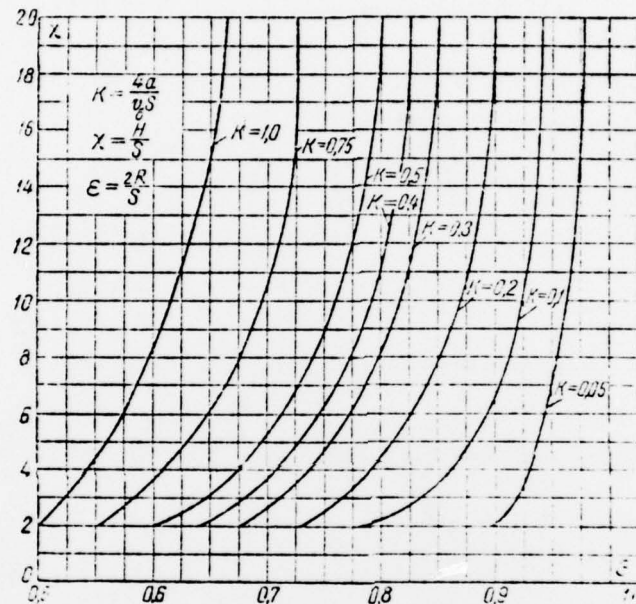


Figure 5. Graph for Determining the Value of Parameter ε at Which the Thermal Influx From the Unfrozen Water-Permeable Ground to the Frozen Ground Columns Reaches Its Peak.

Conclusions

In this article we have examined the problems of making a thermal engineering calculation of water-permeable ground being frozen by a row of columns before the frozen ground cylinders join where the filtration flow arises through the cofferdam under the action of a constant head.

As a consequence a method corresponding to this case has been developed for calculating the period of freezing. In this process it was found that as the frozen ground cylinders increased in size, the thermal influx from the unfrozen ground first increased and then, due to the reduction in the flow rate of the filtration flow, decreased. Therefore the most difficult period of freezing occurs not directly prior to the joining of the frozen ground cylinders, but somewhat beforehand. If by the time the maximum thermal influx from

the filtration flow occurs the frozen ground cylinders are still growing in size, then it is considered that their joining into a unified frozen ground mass is ensured.

It should be noted that the method proposed in this work for a thermal calculation of the cooling of a filtration flow flowing between two frozen ground cylinders may also be used to solve a number of other thermal problems in which the temperature conductivity of a body and its dimensions vary according to a known law (for instance, in the problems of water course thermics). In this process, however, it must be kept in mind that this method is correct only when there are constant temperatures at the boundaries and if the physics of the event under consideration makes it possible to consider that the change in wall thickness (or water course depth, diameter of the penstock, etc.) does not per se cause temperature changes at compatible points.

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